Using unstable periodic orbits to overcome distortion in chaotic signals

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Proposals to use chaos for communications have been hindered by the fact that broadband chaotic signals are distorted by narrow band or frequency dependent communications channels. I show in this paper how the unstable periodic orbits from a chaotic attractor may be used to estimate the parameters of a filter that has acted on a signal from that attractor and estimate the chaotic signal, even when additive noise larger than the chaotic signal is present. [S1063-651X(99)13111-8]

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INTRODUCTION

The broadband nature of chaotic signals has led many researchers to consider them as a new type of signal for communications [1–9]. While the broadband nature of chaotic signals is attractive for applications in communications, it is also a drawback. Broadband signals are sensitive to communications channels with narrow bandwidth or frequency dependent distortion, especially if the distortion is changing in time. In measuring signals from chaotic experiments, the bandwidth of the instrumentation is also important. There has been some work on either narrowing the bandwidth of chaotic signals [10,11] or using adaptive filters to compensate for varying channel characteristics [9,12]. These methods use self-synchronizing chaotic systems, so they are fairly sensitive to additive noise.

In this paper, I describe a method of correcting for channel distortions that is less sensitive to added noise. This method does not use chaotic synchronization, but instead depends on estimating the original chaotic signal by comparing to a set of possible fixed-length sequences built up from the unstable periodic orbits (UPO's) for the chaotic attractor [13,14]. The method described in this paper does require a large amount of computation, so it may not yet be truly practical for communications, but there may be ways to speed up the computation.

UPO's AND SIGNAL ESTIMATION

While there are an infinite number of UPO's embedded in a chaotic attractor, many dynamical quantities may be estimated based on a finite number of low period orbits [15–22]. Thus a finite set of UPO's may be used to construct an approximate skeleton for the attractor. A set of UPO's may be extracted from a time series signal generated by the system [19,23] or calculated directly if the equations of motion are available [24].

GENERAL APPROACH

I would like to construct approximate trajectories for a chaotic attractor from the set of UPO's that have been found for that attractor. I combine UPO's (or parts of UPO's) to form a set of UPO sequences of some fixed length. I use a continuity condition to decide which UPO may follow which other UPO; the beginning of one UPO must come within some small Euclidean distance ε of the end of the previous UPO. The length of the UPO sequence may be shorter than the length of some of the UPO's.

There may be many possible UPO sequences for a particular length, but most sequences never show up in the actual attractor, so a large number of possible sequences can be discarded. I then compare each of these sequences to a fixed length segment of a time series from the chaotic attractor. Different chaotic systems become uncorrelated with each other, so I compare each unstable periodic orbit sequence with the time series segment by taking the cross-correlation between them. I take the UPO sequence with the highest cross-correlation as the best approximation to the segment from the chaotic time series. The fit is never exact because this is only an approximation technique, but, with a large enough set of UPO sequences, I can obtain maximum cross-correlation values of 0.98 or greater.

CORRECTING FOR DISTORTION

In previous work [13,14] these UPO sequences were used to estimate chaotic time series from flows or maps that were coorrupted by noise. In this work, the UPO sequences are used to extract information from a chaotic signal that has been filtered by a second order bandpass filter. The original chaotic signal is encoded with binary information by multiplying it by ± 1. The signal is passed through a digital version of a second order bandpass filter. The signal is then compared with a set of UPO sequences which are filtered with a similar bandpass filter. It is assumed that the parameters of the bandpass filter are not known, so as each UPO sequence is compared to the incoming filtered chaotic signal, the filter parameters are varied to maximize the cross correlation between the filtered UPO sequence and the incoming signal. Once again, the UPO sequence with the largest cross-correlation is taken as the best approximation to the incoming signal, and the filter parameters estimated for that sequence are taken as the best estimate for the filter. The information signal (± 1) may then be estimated based on whether or not the largest cross-correlation is positive or negative.

Fitting the filter parameters is time consuming, but it does not have to be done for each time interval. Depending on

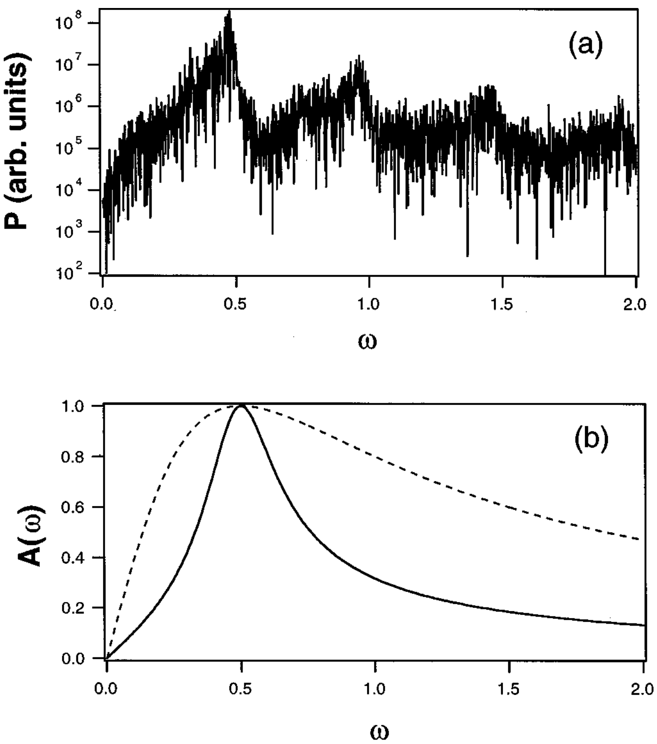

FIG. 1. (a) Power spectrum P for a long time series of the unmodulated z signal from Eq. (1). (b) Amplitude of the transfer function A from Eq. (2). The dotted line is A for $Q=0.5$ and $\omega_c=0.5$; the solid line is for $Q=2.0$ and $\omega_c=0.5$.

how fast the channel is changing, the filter parameters need to be fit only occasionally.

CHAOTIC SYSTEM

The chaotic system used here is a modified version of Sprott's system B [25]. The equations for this system were

$$\frac{dx}{dt}=0.4yz,$$

$$\frac{dy}{dt}=x-1.2y,$$

$$\frac{dz}{dt}=1-xy. \tag{1}$$

The equations were integrated with a fourth order Runge-Kutta integrator with a time step of 0.2 s [26].

The z signal was multiplied by $s=\pm 1$ to create the modulated carrier signal z_s. The modulated carrier signal was then filtered by a digital filter with the transfer function

$$A(\omega)=\frac{i\Omega}{Q(1+i\Omega/Q-\Omega^2)}, \tag{2}$$

where the normalized frequency $\Omega=\omega/\omega_c$, ω_c was the center frequency of the pass band, and Q was the ratio of center frequency to bandwidth. Figure 1(a) shows the power spectrum from a long time series of the z signal (no modulation) from Eq. (1). Figure 1(b) shows the amplitude of the transfer function A. The dotted line shows the transfer function for $Q=0.5$, while the solid line shows the transfer function for $Q=2.0$ ($\omega_c=0.5$ in both cases). The transfer function was used to calculate the coefficients for the digital filter. The filtered signal was z_f. Figure 2 shows the information signal s, the modulated carrier signal z_s, and the filtered carrier signal z_f when $Q=2$ and $\omega_c=0.5$. The value of s was switched every 256 points (about four cycles).

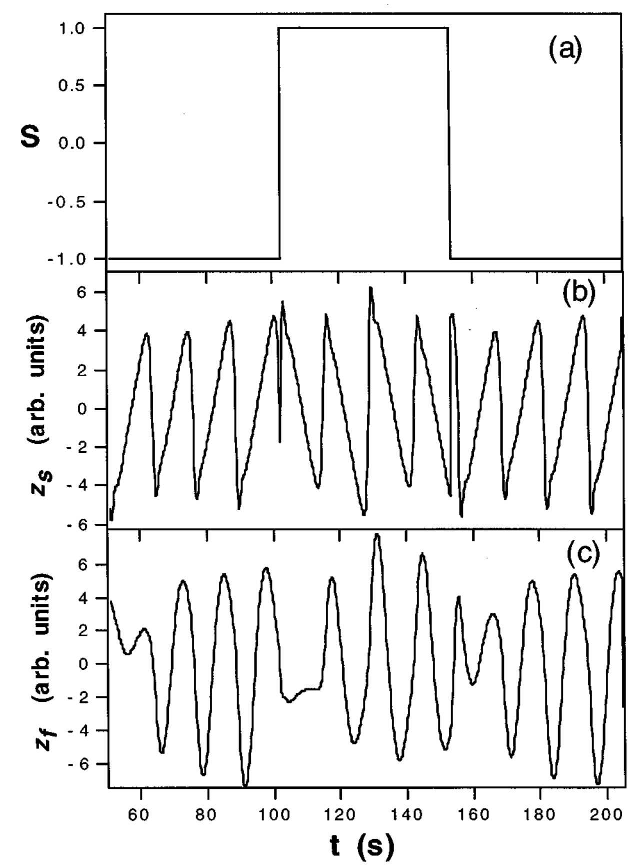

FIG. 2. (a) Information signal s, which modulates the chaotic carrier. (b) Modulated chaotic carrier signal z_s (before filtering). (c) Filtered modulated chaotic carrier signal z_f ($Q=2.0$ and $\omega_c=0.5$).

ESTIMATING THE CARRIER SIGNAL

I first used the method of close approaches [19] to find a set of UPO's up to period 10 for Sprott's system B. The orbits were extracted from a three-dimensional time series. I then constructed a set of UPO sequences about two cycles long (or 128 points at 0.2 s/point). By one cycle (or one period), I mean the period of the lowest period UPO. When creating the UPO sequences, the maximum Euclidean distance allowed between the end of one UPO and the beginning of the next was 2% of the rms amplitude of the orbit. In order to reduce the number of 128-point UPO sequences, I generated a long time series of the z signal from Sprott's system B and for each 128-point segment of the time series found the UPO sequence with the largest cross-correlation.

I kept track of how often each UPO sequence was the best approximation to a segment of the chaotic time series. I found that keeping only the sequences that occurred more

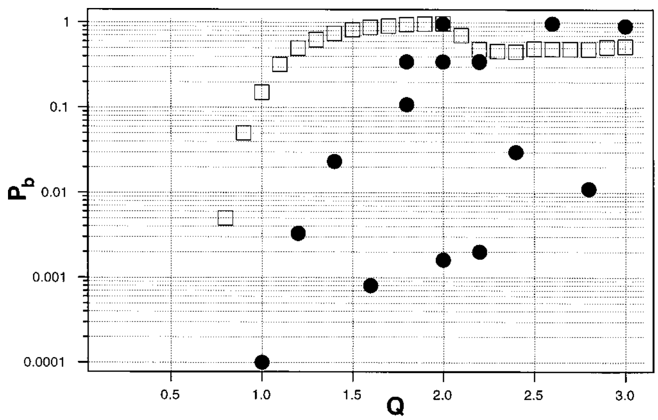

FIG. 3. Probability of the bit error P_b as a function of filter Q. The open squares are when no correcting filter is used on the UPO sequences in the receiver; the solid circles are when the UPO sequences are filtered with a filter whose parameters are adaptively determined.

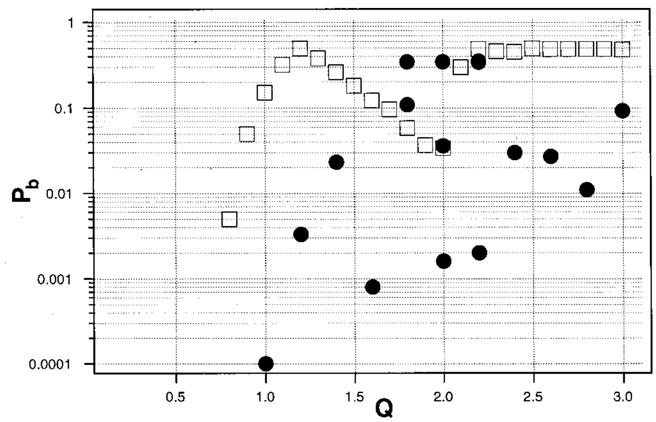

FIG. 4. The same data as Fig. 3, but points over 0.5 have been subtracted from 1.

than 0.05% of the time provided an adequate approximation in that smallest of the maximum cross-correlation values was still greater than 0.9. Keeping more sequences could provide a better approximation.

After truncating the set of 128-point UPO sequences, I combined the remaining sequences to form UPO sequences of length 256 points. I truncated this set of length 256 sequences to obtain a set of 1162 UPO sequences.

To start the process of decoding the information signal, each of these UPO sequences in turn was filtered by a digital filter with a transfer function given by Eq. (2). When detecting the first segment of the time series of z_f, a series of 512 points was passed through the digital filter. The first 256 points were 0, while the next 256 points came from one of the UPO sequences. For later segments of z_s, the first 256 points of the filtered time series came from the UPO sequence corresponding to the best fit to the previous segment of z_s (multiplied by the estimated value of s, ± 1). The second 256 points of the filtered time series came from one of the 1162 UPO sequences. The sequences were filtered in pairs to account for leakage of the previous sequence into the next sequence, caused by the filtering.

The UPO sequences were searched as follows: first, one of the UPO sequences was chosen and filtered. The cross correlation between the filtered UPO sequence and an equal length segment from z_f was calculated for all possible lags, and the largest value of the cross-correlation was saved. The parameters ω_c and Q were then varied by a linear optimization routine [26] in order to maximize the largest cross-correlation between the UPO sequence and z_f. This procedure was repeated for each UPO sequence. The sequence with the largest cross correlation was taken as the best approximation to z_f, and the filter parameters that maximized the cross-correlation for this sequence were taken as the parameters of the filter.

Once the filter parameters had been estimated for the initial 256-point segment of z_f, it was not necessary to estimate them again for the next segment. If the channel parameters do not change too quickly, it is not necessary to perform this estimation very often. If the channel parameters are changing, one could monitor the cross-correlation between z_f and the best UPO sequence, and recalculate the filter parameters if the cross-correlation drops by more than a certain amount.

Other adaptive channel equalization methods attempt to invert the filtering characteristics of the channel [9,12]. Inverting the filter would work here, but would also make the estimation process more sensitive to noise.

Figure 3 shows the result of the detection process as the Q of the filter that produces z_f is varied. The filter center frequency ω_c is fixed at 0.5. The black circles show the probability that an error is made in estimating the value of the information signal s (P_b) when both filter parameters are estimated as described above. The probability of making a bit error becomes larger as the filter Q increases (the bandwidth becomes narrower). For comparison, the open squares in Fig. 3 show the probability of bit error when no filter is used on the UPO sequences in the receiver. The probability of bit error is higher by as much as two orders of magnitude when no filtering is used at the receiver. Clearly, the adaptive filtering at the receiver improves the reception of the chaotic carrier signal.

The highest probability of bit error for a binary scheme should be 0.5; either + or − is equally possible. The probability of bit error in Fig. 3 is seen to exceed 0.5. This means that the receiver is estimating that the bit present is the op-

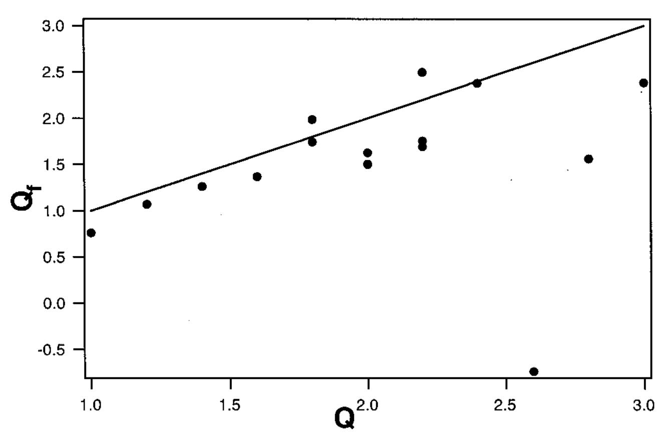

FIG. 5. Adaptively determined filter parameter Q_f as a function of the actual filter parameter Q. The line represents a perfect fit, $Q_f = Q$.

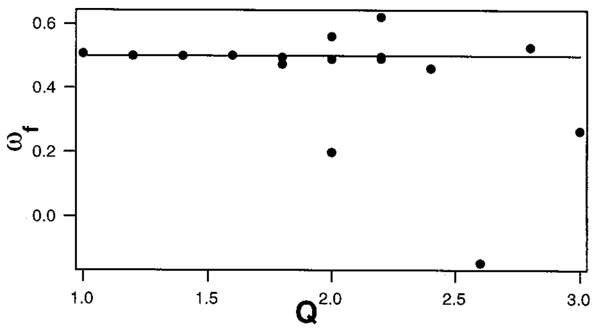

FIG. 6. Adaptively determined filter parameter ω_f as a function of the actual filter parameter Q. The line represents the actual filter parameter $\omega_c = 0.5$.

posite of what was sent. It should be possible to come up with a communications scheme that is not affected by this phase flip (the carrier signal itself could be inverted during transmission), so estimating the opposite bit is not really an error. Figure 4 shows the same information as Fig. 3, but all probabilities over 0.5 are subtracted from 1. For the unfiltered case, the probability of bit error actually goes through a maximum near $Q = 1.2$. The maximum probably comes because different frequencies in the carrier are being phase shifted by different amounts as the filter Q changes. At some phase shift, it is not possible to tell if the carrier signal is rightside up or upside down, but shifting the phase farther makes the carrier appear inverted.

In Figs. 3 and 4, there is considerable scatter in the probability of bit error for the adaptively filtered signal. This is caused by inaccuracy in estimating the filter parameters. Figure 5 shows the estimated Q (Q_f) for the filter vs the actual Q. The solid line is at 45° ($Q_f = Q$). The estimated Q is close to the actual Q for smaller values of Q, but there is considerable departure at larger values.

Figure 6 shows the estimated value of ω_c (ω_f) as a function of the filter Q. The solid line is the actual value of ω_c, 0.5. Once again, as Q increases, there is considerable departure of ω_f from the actual value of ω_c.

EFFECTS OF NOISE

In order to be useful, a communications system must work in the presence of noise. The method described above was evaluated when Gaussian white noise was added to the modulated chaotic time series z_f before filtering. The modulation was detected as above by computing the cross-correlation with possible UPO sequences and fitting the filter parameters from a model of the filter. Figure 7 shows the probability of bit error P_b as a function of the energy per bit E_b divided by the noise power spectral density N_0. The energy per bit is found by multiplying the average signal power by the length of time required to send one bit. The arrow on the E_b/N_0 axis in Fig. 7 corresponds to an unfiltered noise rms equal to the unfiltered signal rms. The filter Q was set at a constant value of 1.0, while the filter frequency $\omega = 0.5$.

The solid circles in Fig. 7 represent the probability of bit error when the UPO sequences were filtered with a digital filter whose parameters were estimated by the methods described above. It can be seen that the probability of bit error is lower when the noise rms is less than or equal to the signal rms. The open squares in Fig. 7 represent the probability of bit error when no filter is used to compensate for filtering by the channel. The adaptive filtering described above clearly improves the probability of bit error even when noise as large as the transmitted signal is present. It was also noted that the estimates of the filter parameters were better for lower levels of noise. Using longer sequences will also reduce the probability of bit error, at the expense of requiring more energy to send a bit.

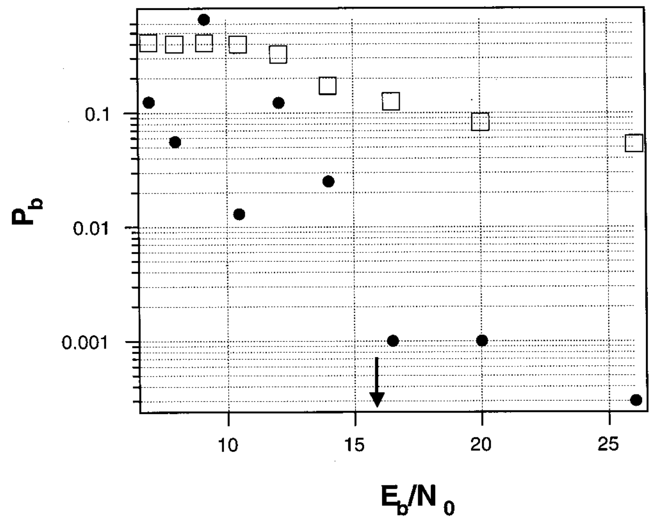

FIG. 7. Probability of the bit error P_b as a function of energy per bit E_b divided by the noise power spectral density N_0 when Gaussian white noise is added to the chaotic carrier. The open squares indicate when no correcting filter is used on the UPO sequences in the receiver, and the solid circles when a correcting filter is used. The arrow indicates where the rms of the unfiltered noise is equal to the rms of the unfiltered carrier signal.

CONCLUSIONS

It was seen previously that it is possible to approximate chaotic time series with UPO sequences. As long as one has already digitized the chaotic signal in order to fit to the UPO sequences, it makes sense to apply other signal processing techniques to improve the received signal. It has been shown here that if the transmission channel acts as a filter, and one has a model of that filter, it is possible to estimate the filter parameters by comparing to the UPO sequences, and use the filter estimate to decrease the probability of bit error in the communication system.

Other than communications, these techniques might also be useful for data analysis, such as identifying the presence of a particular attractor when noise or filtering affects the detection process. It is necessary, of course, to have access to the noise-free attractor beforehand in order to calculate the UPO's. There are computational issues that make the application of the techniques described in this paper to communications difficult. Finding the best fit UPO sequence by calculating cross-correlations with all the other sequences is slow, so some better method of searching for the best sequence is necessary. If such a technique can be found, however, these methods do have much potential for improving communications technology.

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